Discrete Structures I
sample ex 2
Fall 2013

1. Let $M = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

   (a) Evaluate M^2

2. Is it true that if A and B are two 3×3 matrices satisfying $AB = 0$ then one of them must be the zero matrix?Explain..

3. Consider the sequence $\{a_n\}$, where $a_0 = 1$, $a_1 = 2$, $a_2 = 3$ and

$$a_n = a_{n-1} + a_{n-2} + a_{n-3}, \quad n \in \mathbb{Z}^+, \text{ where } n \geq 3$$

   (a) Find a_3, a_4 and a_5.

   (b) Prove by mathematical induction that for all $n \in \mathbb{N}$, we have that $a_n \leq 3^n$

4. Show that $1^2 + 3^2 + 5^2 ... + (2n-1)^2 = \dfrac{n(2n+1)(2n-1)}{3}$ for all n.

5. Let $P(n)$ be the inequality $n^2 < 2^n$.

   (a) Write $P(5), P(k), P(k+1)$

   (b) Show that $P(n)$ holds for all $n \geq 5$. Show all the details

6. As each of a group of business people arrives at a meeting, each shakes hands with all other people present. Use mathematical induction to that if n people come to the meeting, then $\dfrac{n(n-1)}{2}$ hand shakes occur.

7. Prove by mathematical induction that $3 \mid n^3 - n$ for every positive integer n.

8. Consider the function: $f : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \times \mathbb{N}$ given by: $f(m, n) = (3m+n, n^2)$

   (a) Is f one-to-one?

   (b) Is f onto?

9. Show that if $f : S \to T$,and $g : T \to U$ are both 1-1, then $gof : S \to U$ is also 1-1.